Information Hiding—A Survey

Fabien A. P. Petitcolas, Ross J. Anderson and Markus G. Kuhn

Abstract—Information hiding techniques have recently become important in a number of application areas. Digital audio, video, and pictures are increasingly furnished with distinguishing but imperceptible marks, which may contain a hidden copyright notice or serial number or even help to prevent unauthorised copying directly. Military communications systems make increasing use of traffic security techniques which, rather than merely concealing the content of a message using encryption, seek to conceal its sender, its receiver or its very existence. Similar techniques are used in some mobile phone systems and schemes proposed for digital elections. Criminals try to use whatever traffic security properties are provided intentionally or otherwise in the available communications systems, and police forces try to restrict their use. However, many of the techniques proposed in this young and rapidly evolving field can trace their history back to antiquity; and many of them are surprisingly easy to circumvent. In this article, we try to give an overview of the field; of what we know, what works, what does not, and what are the interesting topics for research.

Keywords—Information hiding, steganography, copyright marking.

I. INTRODUCTION

It is often thought that communications may be secured by encrypting the traffic, but this has rarely been adequate in practice. Æneas the Tactician, and other classical writers, concentrated on methods for hiding messages rather than for enciphering them [1]; and although modern cryptographic techniques started to develop during the Renaissance, we find in 1641 that John Wilkins still preferred hiding over ciphering [2] IX pp. 67) because it arouses less suspicion. This preference persists in many operational contexts to this day. For example, an encrypted email message between a known drug dealer and somebody not yet under suspicion, or between an employee of a defence contractor and the embassy of a hostile power, has obvious implications.

So the study of communications security includes not just encryption but also traffic security, whose essence lies in hiding information. This discipline includes such technologies as spread spectrum radio, which is widely used in tactical military systems to prevent transmitters being located; temporary mobile subscriber identifiers, used in digital phones to provide users with some measure of location privacy; and anonymous remailers, which conceal the identity of the sender of an email message [3].

An important subdiscipline of information hiding is steganography. While cryptography is about protecting the content of messages, steganography is about concealing their very existence. It comes from Greek roots (στεγανοσ, γραφεω-ειν), literally means ‘covered writing’ [5], and is usually interpreted to mean hiding information in other information. Examples include sending a message to a spy by marking certain letters in a newspaper using invisible ink, and adding sub-perceptible echo at certain places in an audio recording.

Until recently, information hiding techniques received much less attention from the research community and from industry than cryptography, but this is changing rapidly (table 1), and the first academic conference on the subject was organised in 1996 [1]. The main driving force is concern over copyright; as audio, video and other works become available in digital form, the ease with which perfect copies can be made may lead to large-scale unauthorised copying, and this is of great concern to the music, film, book, and software publishing industries. There has been significant recent research into digital ‘watermarks’ (hidden copyright messages) and ‘fingerprints’ (hidden serial numbers); the idea is that the latter can help to identify copyright violators, and the former to prosecute them.

In another development, the DVD consortium has called for proposals for a copyright marking scheme to enforce serial copy management. The idea is that DVD players available to consumers would allow unlimited copying of home videos and time-shifted viewing of TV programmes, but could not easily be abused for commercial piracy. The proposal is that home videos would be unmarked, TV broadcasts marked ‘copy once only’, and commercial videos marked ‘never copy’; compliant consumer equipment would act on these marks in the obvious way [2].

There are a number of other applications driving interest in the subject of information hiding (figure 1).

- Military and intelligence agencies require unobtrusive communications. Even if the content is encrypted, the detection of a signal on a modern battlefield may lead rapidly to an attack on the signaler. For this reason, military communications use techniques such as spread spectrum modulation or meteor scatter transmission to make signals hard for the enemy to detect or jam.

- Criminals also place great value on unobtrusive communications. Their preferred technologies include prepaid mobile phones, mobile phones which have been modified to change their identity frequently, and hacked corporate switchboards through which calls can be rerouted.

- Law enforcement and counter intelligence agencies are interested in understanding these technologies and their weaknesses, so as to detect and trace hidden messages.

- Recent attempts by some governments to limit online free speech and the civilian use of cryptography have spurred people concerned about liberties to develop techniques for anonymous communications on the net, including anonymous remailers and Web proxies.

- Schemes for digital elections and digital cash make use

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of anonymous communication techniques.

- Marketeers use email forgery techniques to send out huge numbers of unsolicited messages while avoiding responses from angry users.

We will mention some more applications later. For the time being, we should note that while the ethical positions of the players in the cryptographic game are often thought to be clear cut (the ‘good’ guys wish to keep their communications private while the ‘bad’ eavesdropper wants to listen in), the situation is much less clear when it comes to hiding information. Legitimate users of the net may need anonymous communications to contact abuse helplines or vote privately in online elections [8]; but one may not want to provide general anonymous communication mechanisms that facilitate attacks by people who maliciously overload the communication facilities. Industry may need tools to hide copyright marks invisibly in media objects, yet these tools can be abused by spies to pass on secrets hidden in inconspicuous data over public networks. Finally, there are a number of non-competitive uses of the technology, such as marking audio tracks with purchasing information so that someone listening to a piece of music on his car radio could simply press a button to order the CD.

The rest of this paper is organised as follows. Firstly, we will clarify the terminology used for information hiding, including steganography, digital watermarking and fingerprinting. Secondly we will describe a wide range of techniques that have been used in a number of applications, both ancient and modern, which we will try to juxtapose in such a way that the common features become evident. Then, we will describe a number of attacks against these techniques; and finally, we will try to formulate general definitions and principles. Moving through the subject from practice to theory may be the reverse of the usual order of presentation, but appears appropriate to a discipline in which rapid strides are being made constantly, and where general theories are still very tentative.

II. TERMINOLOGY

As we have noted previously, there has been a growing interest, by different research communities, in the fields of steganography, digital watermarking and fingerprinting. This led to some confusion in the terminology. We shall now briefly introduce the terminology which will be used in the rest of the paper and which was agreed at the first international workshop on the subject [1], [2] (figure 1).

The general model of hiding data in other data can be described as follows. The *embedded* data is the message that one wishes to send secretly. It is usually hidden in an innocuous message referred to as a *cover-text*, or *cover-image* or *cover-audio* as appropriate, producing the *stego-text* or other *stego-object*. A *stego-key* is used to control the hiding process so as to restrict detection and/or recovery of the embedded data to parties who know it (or who know some derived key value).

As the purpose of steganography is having a covert communication between two parties whose existence is unknown to a possible attacker, a successful attack consists in detecting the existence of this communication. Copyright marking, as opposed to steganography, has the additional requirement of robustness against possible attacks. In this context, the term ‘robustness’ is still not very clear; it mainly depends on the application. Copyright marks do not always need to be hidden, as some systems use *visible digital watermarks* [11], but most of the literature has focussed on invisible (or transparent) digital watermarks which have wider applications. Visible digital watermarks are strongly linked to the original paper watermarks which appeared at the end of the XIII century to differentiate paper makers of that time [12] (figure 2). Modern visible watermarks may be visual patterns (e.g., a company logo or copyright sign) overlaid on digital images.

In the literature on digital marking, the stego-object is usually referred to as the *marked object* rather than stego-object. We may also qualify marks depending on the application. *Fragile watermarks* [1] are destroyed as soon as the

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Table 1

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**Table 1**

**Number of publications on digital watermarking during the past few years according to INSPEC, January 99. Courtesy of J.-L. Dugelay [5].**

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1 Fragile watermarks have also wrongly been referred to as ‘signature’, leading to confusion with digital signatures used in cryptography.
object is modified too much. This can be used to prove that an object has not been ‘doctored’ and might be useful if digital images are used as evidence in court. Robust marks have the property that it is infeasible to remove them or make them useless without destroying the object at the same time. This usually means that the mark should be embedded in the most perceptually significant components of the object [13].

Authors also make the distinction between various types of robust marks. Fingerprints (also called labels by some authors) are like hidden serial numbers which enable the intellectual property owner to identify which customer broke his license agreement by supplying the property to third parties. Watermarks tell us who is the owner of the object.

Figure 2 illustrates the generic embedding process. Given an image $I$, a mark $M$ and a key $K$ (usually the seed of a random number generator) the embedding process can be defined as a mapping of the form: $I \times K \times M \rightarrow \tilde{I}$ and is common to all watermarking methods.

The generic detection process is depicted in figure 3. Its output is either the recovered mark $M$ or some kind of confidence measure indicating how likely it is for a given mark at the input to be present in the image $\tilde{I}$ under inspection.

There are several types of robust copyright marking systems. They are defined by their inputs and outputs:

- **Private marking** systems require at least the original image. Type I systems, extract the mark $M$ from the possibly distorted image $\tilde{I}$ and use the original image as a hint to find where the mark could be in $\tilde{I}$. Type II systems (e.g., [14, 15, 16]) also require a copy of the embedded mark for extraction and just yield a ‘yes’ or ‘no’ answer to the question: does $\tilde{I}$ contain the mark $M$? ($\tilde{I} \times I \times K \times M \rightarrow \{0, 1\}$). One might expect that this kind of scheme will be more robust than the others since it conveys very little information and requires access to secret material [13]. Semi-private marking does not use the original image for detection ($\tilde{I} \times K \times M \rightarrow \{0, 1\}$) but answers the same question.

The main uses of private and semi-private marking seem to be evidence in court to prove ownership and copy control in applications such as DVD where the reader needs to know whether it is allowed to play the content or not. Many of the currently proposed schemes fall in this category [17, 18, 19, 20, 21, 22, 23].

- **Public marking** (also referred to as blind marking) remains the most challenging problem since it requires neither the secret original $I$ nor the embedded mark $M$. Indeed such systems really extract $n$ bits of information (the mark) from the marked image: $\tilde{I} \times K \rightarrow M$ [24, 25, 26, 27, 28]. Public marks have much more applications than the others and we will focus our benchmark on these systems. Indeed the embedding algorithms used in public systems can usually be used in private ones, improving robustness at the same time.

\[ Mark (M) \rightarrow \text{Stego-image} \rightarrow \text{Marking algorithm} \rightarrow \text{Marked image} \]

\[ \text{Mark or confidence measure} \]

- There is also asymmetric marking (or public key marking) which should have the property that any user can read the mark, without being able to remove it.

In the rest of the paper, ‘watermark’ will refer to ‘digital watermark’ unless said otherwise.

### III. Steganographic techniques

We will now look at some of the techniques used to hide information. Many of these go back to antiquity, but unfortunately many modern system designers fail to learn from the mistakes of their predecessors.

#### A. Security through obscurity

By the 16–17th centuries, there had arisen a large literature on steganography and many of the methods depended on novel means of encoding information. In his four hundred page book *Schola Steganographica* [29], Gaspar Schott (1608–1666) explains how to hide messages in music scores: each note corresponds to a letter (figure 4). Another method, based on the number of occurrences of notes and used by J. S. Bach, is mentioned in [10]. Schott also expands the ‘Ave Maria’ code proposed by Johannes Trithemius (1462–1516) in *Steganographia*, one of the first known books in the field. The expanded code uses forty tables, each of which contains 24 entries (one for each letter of the alphabet of that time) in four languages: Latin, German, Italian and French. Each letter of the plain-text is replaced by the word or phrase that appears in the corresponding table entry and the stego-text ends up looking like a prayer or a magic spell. It has been shown recently that these tables can be deciphered by reducing them modulo 25 and applying them to a reversed alphabet [30]. In [2], John Wilkins (1614–1672), Master of Trinity College Cambridge, shows how ‘two Musicians may discourse with one another by playing upon their instruments of musick as well
Hiding information into music scores: Gaspar Schott simply maps the letters of the alphabet to the notes. Clearly, one should not try to play the music [29, p. 322].

As by talking with their instruments of speech [2] XVIII, pp. 143–150. He also explains how one can hide secretly a message into a geometric drawing using points, lines or triangles. ‘The point, the ends of the lines and the angles of the figures do each of them by their different situation express a several letter’ [2, XI, pp. 88–96].

A very widely used method is the acrostic. In his book, The Codebreakers [31], David Kahn explains how a monk wrote a book and put his lover’s name in the first letters of The Codebreakers [31], David Kahn explains how a monk wrote a book and put his lover’s name in the first letters of The Codebreakers.

He also tells of prisoners of war who wrote a book and put his lover’s name in the first letters of successive chapters. He also tells of prisoners of war who wrote a book and put his lover’s name in the first letters of successive chapters.

The situation may be improved by intelligent use of camouflage. Even if the method is known in principle, making the hidden data expensive to look for can be beneficial, especially where there is a large amount of cover traffic.

During both World Wars, censors intercepted many such messages. A famous one, from World War I, was a cablegram saying ‘Father is dead’ which the censor modified into ‘Father is deceased’. The reply was a giveaway: ‘Is Father dead or deceased?’ [31, pp. 515–516].

Although steganography is different from cryptography, we can borrow many of the techniques and much practical wisdom from the latter, more thoroughly researched discipline. In 1883, Auguste Kerckhoffs enunciated the first principles of cryptographic engineering, in which he advises that we assume the method used to encipher data is known to the opponent, so security must lie only in the choice of key. The history of cryptography since then has repeatedly shown the folly of ‘security-by-obscurity’ – the assumption that the enemy will remain ignorant of the system in use.

Applying this wisdom, we obtain a tentative definition of a secure stego-system: one where an opponent who understands the system, but does not know the key, can obtain no evidence (or even grounds for suspicion) that a communication has taken place. In other words, no information about the embedded text can be obtained from knowledge of the stego (and perhaps also cover) texts. We will revisit this definition later, to take account of robustness and other issues; but it will remain a central principle that steganographic processes intended for wide use should be published, just like commercial cryptographic algorithms and protocols. This teaching of Kerckhoffs holds with particular force for marking techniques intended for use in evidence, which implies their disclosure in court [33].

B. Camouflage

The situation may be improved by intelligent use of camouflage. Even if the method is known in principle, making the hidden data expensive to look for can be beneficial, especially where there is a large amount of cover traffic.

Since the early days of architecture, artists have understood that works of sculpture or painting appear different from certain angles, and established rules for perspectival and anamorphosis [34]. Through the 16th and 17th centuries anamorphic images supplied an ideal means of camouflaging dangerous political statements and heretical ideas [35]. A masterpiece of hidden anamorphic imagery – the Vexierbild – was created in the 1530s by Shō, a Nürnberg engraver, pupil of Albrecht Dürer (1471–1528): when one looks at it normally one sees a strange landscape, but looking from the side reveals portraits of famous kings.

In his Histories [36], Herodotus (c.486–425 B.C.) tells how around 440 B.C. Histiaeus shaved the head of his most trusted slave and tattooed it with a message which disappeared after the hair had regrown. The purpose was to instigate a revolt against the Persians. Astonishingly the method was still used by some German spies at the beginning of the 20th century [37]. Herodotus also tells how Demeratus, a Greek at the Persian court, warned Sparta of an imminent invasion by Xerxes: he removed the wax from a writing tablet, wrote his message on the wood underneath and then covered the message with wax. The tablet looked exactly like a blank one (it almost fooled the recipient as well as the customs men).

A large number of techniques were invented or reported by Æneas the Tactician [1], including letters hidden in messengers’ soles or women’s ear-rings, text written on wood tablets and then whitewashed, and notes carried by pigeons. The centerpiece is a scheme for winding thread through 24 holes bored in an astragal: each hole represents a letter and a word is represented by passing the thread through the corresponding letters. He also proposed hiding text by making very small holes above or below letters or by changing the heights of letter-strokes in a cover text. These dots were masked by the contrast between the black letters and the white paper. This technique was still in use during the 17th century, but was improved by Wilkins
who used invisible ink to print very small dots instead of making holes and was reused by German spies during both World Wars. A modern adaptation of this technique is still in use for document security.

Invisible inks were used extensively. They were originally made of available organic substances (such as milk or urine) or ‘salt armoniack dissolved in water’ and developed with heat; progress in chemistry helped to create more sophisticated combinations of ink and developer by the first World War, but the technology fell into disuse with the invention of ‘universal developers’ which could determine which parts of a piece of paper had been wetted from the effects on the surfaces of the fibres. Nowadays, in the field of currency security, special inks or materials with particular structure (such as fluorescent dyes or DNA) are used to write a hidden message on bank notes or other secure documents. These materials provide a unique response to some particular excitation such as a reagent or laser light at a particular frequency.

By 1860 the basic problems of making tiny images had been solved. In 1857, Brewster suggested hiding secret messages ‘in spaces not larger than a full stop or small dot’ by pigeon post. During the Russo-Japanese war of 1905, microscopic images were hidden in ears, nostrils, and under finger nails. By World War I messages to and from spies were reduced to microdots by several stages of photographic reduction and then stuck on top of printed periods or commas in innocuous cover material such as magazines.

The digital equivalent of these camouflage techniques is the use of masking algorithms. Like most source-coding techniques, these rely on the properties of the human perceptual system. Audio masking, for instance, is a phenomenon in which one sound interferes with our perception of another sound. Frequency masking occurs when two tones which are close in frequency are played at the same time: the louder tone will mask the quieter one. Temporal masking occurs when a low-level signal is played immediately before or after a stronger one; after a loud sound stops, it takes a little while before we can hear a weak tone at a nearby frequency.

Because these effects are used in compression standards such as MPEG, many systems shape the embedded data to emphasise it in the perceptually most significant components of the data so it will survive compression (figure 5). This idea is also applied in buried data channels where the regular channels of an audio CD contain other embedded sound channels; here, an optimised noise shaper is used to reduce the effect of the embedded signal on the quality of the cover music.

For more details about the use of perceptual models in digital watermarking, the reader is referred to.

C. Hiding the location of the embedded information

In a security protocol developed in ancient China, the sender and the receiver had copies of a paper mask with a number of holes cut at random locations. The sender would place his mask over a sheet of paper, write the secret message into the holes, remove the mask and then compose a cover message incorporating the code ideograms. The receiver could read the secret message at once by placing his mask over the resulting letter. In the early 16th century Cardan (1501–1576), an Italian mathematician, reinvented this method which is now known as the Cardan grille. It appears to have been reinvented again in 1992 by a British bank, which recommended that its customers conceal the personal information number used with their cash machine card using a similar system. In this case, a poor implementation made the system weak.

A variant on this theme is to mark an object by the presence of errors or stylistic features at predetermined points in the cover material. An early example was a technique used by Francis Bacon (1561–1626) in his biliterarie alphabet, which seems to be linked to the controversy whether he wrote the works attributed to Shakespeare. In this method each letter is encoded in a five-bit binary code and embedded in the cover-text by printing the letters in either normal or italic fonts. The variability of sixteenth century typography acted as camouflage.

Further examples come from the world of mathematical tables. Publishers of logarithm tables and astronomical ephemerides in the 17th and 18th century used to introduce errors deliberately in the least significant digits. To this day, database and mailing list vendors insert bogus entries in order to identify customers who try to resell their products.

In an electronic publishing pilot project copyright messages and serial numbers have been hidden in the line spacing and other format features of documents. It was found that shifting text lines up or down by one-three-
hundredth of an inch to encode zeros and ones was robust against multi-generation photocopying and could not be noticed by most people.

However, the main application area of current copyright marking proposals, lies in digital representations of analogue objects such as audio, still pictures, video and multimedia generally. Here there is considerable scope for embedding data by introducing various kinds of error. As we noted above, many writers have proposed embedding the data in the least significant bits of flat color, or lying on a sharp edge; for this reason, a sequence generator to select candidate pixels for embedding is needed. The simplest is the repetition code, and to use suitable coding techniques to exploit the residual bandwidth. The simplest is the repetition code. If $I$ is the original image and $\tilde{I}$ the watermarked image, that is the image whose main components have been modified, the presence of the watermark is verified by extracting the main components of $\tilde{I}$ and those with same index from $I$ and inverting the embedding formula to give a possibly modified watermark $W'$. The watermark is said to be present in $I$ if the ratio $W \cdot W'/\sqrt{W \cdot W'}$ is greater than a given threshold.

The authors claim that $O(\sqrt{n} \ln n)$ similar watermarks must be added before they destroy the original mark. This method is very robust against rescaling, JPEG compression, dithering, clipping, printing/scanning, and collusion attacks. However it has some drawbacks. Most seriously, the original image is needed to check for the presence of a watermark.

The second problem is the low information rate. Like Patchwork, this scheme hides a single bit and is thus suitable for watermarking rather than fingerprinting or steganographic communication. The information rate of such schemes can again be improved by placing separate marks in the image, but at a cost of reduced robustness.

Information hiding schemes that operate in a transform space are increasingly common, as this can aid robustness against compression, other common filtering operations, and noise. Actually one can observe that the use of a particular transform gives good results against compression algorithms based on the same transform.
Some schemes operate directly on compressed objects (e.g., [72]). Some, steganographic tools, for example, hide information in GIF [74] files by swapping the colours of selected pixels for colours that are adjacent in the current palette [7]. Another example is MP3Stego [76] which hides information in MPEG Audio Layer III bitstreams [50] during the compression process. However, most schemes operate directly on the components of some transform of the cover object like discrete cosine transform [15], [16], [17], [77], [78], [79], wavelet transforms [16], [80], and the discrete Fourier transform [73], [81].

A novel transform coding technique is echo hiding [82], which relies on the fact that we cannot perceive short echoes (of the order of a millisecond). It embeds data into a cover audio signal by introducing two types of short echo with different delays to encode zeros and ones. These bits are encoded at locations separated by spaces of pseudo-random length. The cepstral transform [83] is used to manipulate the echo signals.

E. Techniques specific to the environment

Echo hiding leads naturally to the broader topic of information hiding techniques that exploit features of a particular application environment. One technology that is emerging from the military world is meteor burst communication, which uses the transient radio paths provided by ionised trails of meteors entering the atmosphere to send data packets between a mobile station and a base [84]. The transient nature of these paths makes it hard for an enemy to locate mobiles using radio direction finding, and so meteor burst is used in some military networks.

More familiar application-specific information hiding and marking technologies are found in the world of security printing. Watermarks in paper are a very old anti-counterfeiting technique (figure 6); more recent innovations include special ultra-violet fluorescent inks used in printing travellers’ cheques. As the lamps used in photocopiers have a high UV content, it can be arranged that photocopied text becomes void in large letters. Inks may also be reactive; one of the authors has experience of travelling in an American vehicle where the carbon disulfide in the ink was neutralised by a chemical reaction after exposure to perspiration in a money belt. Recent developments address the problem of counterfeiting with scanners and printers whose capabilities have improved dramatically over the last few years [85].

Many other techniques are used. For a survey of optically variable devices, such as diffraction products and thin film interference coatings, see [86]; the design of the US currency is described in [88], [89]; and the security features of the Dutch passport in [90]. Such products tend to combine overt marks that are expensive to reproduce (holograms, kinegrams, intaglios and optically variable inks) with tamper-evidence features (such as laminates and reactive inks) and secondary features whose presence may not be obvious (such as micro-printing, diffraction effects visible only under special illumination, and alias band structures – dithering effects that normal scanners cannot capture), [91], [92]. In a more recent application called sub-graving, variable information (such as a serial number) is printed on top of a uniform offset background. The printed area is then exposed to an excimer laser: this removes the offset ink everywhere but underneath the toner. Fraudulent removal of the toner by a solvent reveals the hidden ink [93].

Increasingly, features are incorporated that are designed to be verified by machines rather than humans. Marks can be embedded in the magnetic strips of bank cards, giving each card a unique serial number that is hard to reproduce [94]; they are used in phone cards too in some countries. Magnetic fibres can be embedded randomly in paper or cardboard, giving each copy of a document a unique fingerprint.

The importance of these technologies is not limited to protecting currency and securities. Forgery of drugs, vehicle spares, computer software and other branded products is said to have cost over $24 billion in 1995, and to have directly caused over 100 deaths worldwide [95]. Security printing techniques are a significant control measure, although many fielded sealing products could be much better designed given basic attention to simple issues such as choice of pressure-sensitive adhesives and nonstandard materials [96]. Fashion designers are also concerned that their product might be copied and wish to find techniques to enable easy detection of counterfeit clothes or bags. As a greater percentage of the gross world product comes in the form of digital objects, the digital marking techniques described here may acquire more economic significance.
Also important are covert channels: communication paths that were neither designed nor intended to transfer information at all. Common examples include timing variations and error messages in communication protocols and operating system call interfaces [97, 98]. Covert channels are of particular concern in the design and evaluation of mandatory access control security concepts, where the operating system attempts to restrict the flow of information between processes in order to protect the user from computer viruses and Trojan horse software that transmits secrets to third parties without authorization.

The electromagnetic radiation produced by computers forms another covert channel. It not only interferes with reception on nearby radio receivers, but can also convey information. For instance, the video signal emitted by CRT or liquid-crystal displays can be reconstructed using a simple modified TV set at several hundred meters distance [99]. Many military organizations use especially shielded ‘Tempest’ certified equipment to process classified information, in order to eliminate the risk of losing secrets via compromising emanations [100].

We have shown in [101] how software can hide information in video screen content in a form that is invisible to the user but that can easily be reconstructed with modified TV receivers. More sophisticated ways of broadcasting information covertly from PC software use spread spectrum techniques to embed information in the video signal or CPU bus activity.

It is possible to write a virus that searches a computer’s hard disk for crypto-key material or other secrets, and proceeds to radiate them covertly. The same techniques could also be used in software copyright protection: software could transmit its license serial number while in use, and software trade associations could send detector vans round business districts and other neighbourhoods where piracy is suspected – just like the ‘TV detector vans’ used in countries with a mandatory TV license fee. If multiple signals are then received simultaneously with the same serial number but with spreading sequences at different phases, this proves that software purchased under a single license is being used concurrently on different computers, and can provide the evidence to obtain a search warrant.

IV. LIMITATIONS OF SOME INFORMATION HIDING SYSTEMS

A number of broad claims have been made about the ‘robustness’ of various digital watermarking or fingerprinting methods. Unfortunately the robustness criteria and the sample pictures used to demonstrate it vary from one system to the other, and recent attacks [102, 103, 104]. [105], [106] show that the robustness criteria used so far are often inadequate. JPEG compression, additive Gaussian noise, low pass filtering, rescaling, and cropping have been addressed in most of the literature but specific distortions such as rotation have often been ignored [81], [107]. In some cases the watermark is simply said to be ‘robust against common signal processing algorithms and geometric distortions when used on some standard images’. This motivated the introduction of a fair benchmark for digital image watermarking in [108].

Similarly, various steganographic systems have shown serious limitations [109].

Craver et al. [110] identify at least three kinds of attacks: robustness attacks which aim to diminish or remove the presence of a digital watermark, presentation attacks which modify the content such that the detector cannot find the watermark anymore (e.g., the Mosaic attack, see section IV-C) and the interpretation attacks whereby an attacker can devise a situation which prevents assertion of ownership. The separation between these groups is not always very clear though; for instance, StirMark (see section IV-B.1) both diminishes the watermark and distorts the content to fool the detector.

As examples of these, we present in this section several attacks which reveal significant limitations of various marking systems. We will develop a general attack based on simple signal processing, plus specialised techniques for some particular schemes, and show that even if a copyright marking system were robust against signal processing, bad engineering can provide other avenues of attacks.

A. Basic attack

Most simple spread spectrum based techniques and some simple image stego software are subject to some kind of jitter attack [104]. Indeed, although spread spectrum signals are very robust to amplitude distortion and to noise addition, they do not survive timing errors: synchronisation of the chip signal is very important and simple systems fail to recover this synchronisation properly. There are more subtle distortions that can be applied. For instance, in [111], Hamdy et al. present a way to increase or decrease the length of a musical performance without changing its pitch; this was developed to enable radio broadcasters to slightly adjust the playing time of a musical track. As such tools become widely available, attacks involving sound manipulation will become easy.

B. Robustness attacks

B.1 StirMark

After evaluating some watermarking software, it became clear to us that although most schemes could survive basic manipulations – that is, manipulations that can be done easily with standard tools, such as rotation, shearing, resampling, resizing and lossy compression – they would not cope with combinations of them or with random geometric distortions. This motivated the design of StirMark [104].

StirMark is a generic tool for basic robustness testing of image watermarking algorithms and has been freely available since November 1997 [1]. It applies a minor unnoticeable geometric distortion: the image is slightly stretched, sheared, shifted, bent and rotated by an unnoticeable random amount. A slight random low frequency deviation, which is greatest at the centre of the picture, is applied to each pixel. A higher frequency displacement of the form

<http://www.cl.cam.ac.uk/~fapp2/watermarking/stirmark/>
example of these distortions is given in figure 7. the approximating quadratic B-spline algorithm [112]. An example of these distortions is given in figure 7.

\[ \lambda \sin(\omega x) \sin(\omega y) + n(x, y) \] where \( n(x, y) \) is a random number – is also added. Finally a transfer function that introduces a small and smoothly distributed error into all sample values is applied. This emulates the small non-linear analogue/digital converter imperfections typically found in scanners and display devices. Resampling uses the approximating quadratic B-spline algorithm [112]. An example of these distortions is given in figure 7.

StirMark are almost unnoticeable: ‘Lena’ before (a) and after (b) StirMark with default parameters. For comparison, the same distortions have been applied to a grid (c & d).

StirMark can also perform a default series of tests which serve as a benchmark for image watermarking [108]. Digital watermarking remains a largely untested field and very few authors have published extensive tests on their systems (e.g., [113]). A benchmark is needed to highlight promising areas of research by showing which techniques work better than others.

One might try to increase the robustness of a watermarking system by trying to foresee the possible transforms used by pirates; one might then use techniques such as embedding multiple versions of the mark under suitable inverse transforms; for instance O’Ruanaidh et al. [81] suggest using the Fourier-Mellin transform.

However, the general lesson from this attack is that given a target marking scheme, one can invent a distortion (or a combination of distortions) that will prevent detection of the watermark while leaving the perceptual value of the previously watermarked object undiminished. We are not limited in this process to the distortions produced by common analogue equipment, or usually applied by end users with common image processing software. Moreover, the quality requirements of pirates are often lower than those of content owners who have to decide how much quality degradation to tolerate in return for extra protection offered by embedding a stronger signal. It is an open question whether there is any digital watermarking scheme for which a chosen distortion attack cannot be found.

B.2 Attack on echo hiding

As mentioned above, echo hiding encodes zeros and ones by adding echo signals distinguished by two different values for their delay \( \tau \) and their relative amplitude \( \alpha \) to a cover audio signal. The delays are chosen between 0.5 and 2 ms, and the relative amplitude is around 0.8 [82]. According to its creators, decoding involves detecting the initial delay and the auto-correlation of the cepstrum of the encoded signal is used for this purpose. However the same technique can be used for an attack.

The ‘obvious’ attack on this scheme is to detect the echo and then remove it by simply inverting the convolution formula; the problem is to detect the echo without knowledge of either the original object or the echo parameters. This is known as ‘blind echo cancellation’ in the signal processing literature and is known to be a hard problem in general.

We tried several methods to remove the echo. Frequency invariant filtering [114], [115] was not very successful. Instead we used a combination of cepstrum analysis and ‘brute force’ search.

The underlying idea of cepstrum analysis is presented in [83]. Suppose that we are given a signal \( y(t) \) which contains a simple single echo, i.e. \( y(t) = x(t) + \alpha x(t - \tau) \). If \( \Phi_{xx} \) denotes the power spectrum of \( x \) then \( \Phi_{yy}(f) = \Phi_{xx}(f)(1 + 2\alpha \cos(2\pi f \tau) + \alpha^2) \) whose logarithm \( \log \Phi_{yy}(f) \approx \log \Phi_{xx}(f) + 2\alpha \cos(2\pi f \tau) \). Taking its power spectrum raises its ‘quefrency’ \( \tau \), that is the frequency of \( \cos(2\pi f \tau) \) as a function of \( f \). The auto-covariance of this later function emphasises the peak that appears at ‘quefrency’ \( \tau \).

We need a method to detect the echo delay \( \tau \) in a signal. For this, we used a slightly modified version of the cepstrum: \( C \circ \Phi \circ \ln \circ \Phi \), where \( C \) is the auto-covariance function \( C(x) = E((x - \bar{x})(x - \bar{x})^*) \), \( \Phi \) the power spectrum density function and \( \circ \) the composition operator. Experiments on random signals as well as on music show that this method returns quite accurate estimators of the delay when an artificial echo has been added to the signal. In the detection function we only consider echo delays between 0.5 and 3 ms (below 0.5 ms the function does not work properly and above 3 ms the echo becomes too audible).

Our first attack was to remove an echo with random relative amplitude, expecting that this would introduce enough modification in the signal to prevent watermark recovery. Since echo hiding gives best results for \( \alpha \) greater than 0.7 we could use \( \hat{\alpha} \) – an estimator of \( \alpha \) – drawn from, say a normal distribution centred on 0.8. It was not really successful
so our next attack was to iterate: we re-applied the detection function and varied α to minimise the residual echo. We could obtain successively better estimates of the echo parameters and then remove this echo. When the detection function cannot detect any more echo, we have found the correct value of α (as this gives the lowest output value of the detection function).

B.3 Other generic attacks

Some generic attacks attempt to estimate the watermark and then remove it. Langelaar et al. [105], for instance, present an attack on white spread spectrum watermarks. They try different methods to model the original image and apply this model to the watermarked image \( I = I + W \) to separate it into two components: an estimated image \( \hat{I} \) and an estimated watermark \( \hat{W} \). They observe that the watermark \( W \) does not appear anymore in \( \hat{I} \), giving \( \rho(I, W) \approx 0 \). The authors show that a 3x3 median filter gives the best results. However an amplified version of the estimated watermark needs to be subtracted because the low frequency components of the watermark cannot be estimated accurately, leading to a positive contribution of the low frequencies and a negative contribution of the high frequencies to the correlation. Only a choice of good amplification parameters can zero the correlation.

In some cases the image to be marked has certain features that help a malicious attacker to gain information about the mark itself. An example of such features is where a picture, such as a cartoon, has only a small number of distinct colours, giving sharp peaks in the colour histogram. These are split by some marking algorithms. The twin peaks attack, suggested by Maes [103], takes advantage of this to recover and remove marks. In the case of grayscale images, a simple example of digital watermarking based on spread spectrum ideas is to add or subtract randomly a fixed value \( d \) to each pixel value. So each pixel’s value has a 50% chance of being increased or decreased. Let \( n_k \) be the number of pixels with gray value \( k \) and suppose that for a particular gray value \( k_0 \) the \( d \)th neighbouring colours do not occur, so \( n_{k_0} = n_{k_0+d} = 0 \). Consequently, the expected numbers of occurrences after watermarking are: \( \hat{n}_{k_0} = \hat{n}_{k_0+d} = n_{k_0}/2 \) and \( \hat{n}_{k_0} = 0 \). Hence, using a set of similar equations, it is possible in certain cases, to recover the original distribution of the histogram and the value of the embedded watermark.

C. The mosaic attack

There is a presentation attack which is quite general and which possesses the initially remarkable property that we can remove the marks from an image and still have it rendered exactly the same, pixel for pixel, as the marked image by a standard browser.

It was motivated by a fielded system for copyright piracy detection, consisting of a watermarking scheme plus a web crawler that downloads pictures from the net and checks whether they contain a client’s watermark.

Our mosaic attack consists of chopping an image up into a number of smaller subimages, which are embedded one after another in a web page. Common web browsers render juxtaposed subimages stuck together as a single image, so the result is identical to the original image. This attack appears to be quite general: all marking schemes require the marked image to have some minimal size (one cannot hide a meaningful mark in just one pixel). Thus by splitting an image into sufficiently small pieces, the mark detector will be confused [104]. One defence would be to ensure that the minimal size would be quite small and the mosaic attack might therefore not be very practical.

But there are other problems with such ‘crawlers’. Mobile code such as Java applets can be used to display a picture inside the browser; the applet could de-scramble the picture in real time. Defeating such techniques would entail rendering the whole page, detecting pictures and checking whether they contain a mark. Another problem is that pirated pictures are typically sold via many small web services, from which the crawler would have to purchase them using a credit card before it could examine them.

D. Interpretation attacks

StirMark and our attack on echo hiding are examples of the kind of threat that dominates the information hiding literature – namely, a pirate who removes the mark directly using technical means. Indeed, the definition commonly used for robustness includes only resistance to signal manipulation (cropping, scaling, resampling, etc.). However, Craver et al. show that this is not enough by exhibiting a ‘protocol’ level attack in [116].

The basic idea is that as many schemes provide no intrinsic way of detecting which of two watermarks was added first. If the owner of the document \( d \) encodes a watermark \( w \), publishes the marked version \( d + w \) and has no other proof of ownership, then a pirate who has registered his watermark as \( w' \) can claim that the document is his and that the original unmarked version of it was \( d + w - w' \). Their paper [117] extends this idea to defeat a scheme which is non-invertible (an inverse needs only be approximated).

Craver et al. argue for the use of information-losing marking schemes whose inverses cannot be approximated closely enough. Our alternative interpretation of their attack is that watermarking and fingerprinting methods must be used in the context of a larger system that may use mechanisms such as timestamping and notarisation to prevent attacks of this kind.

Environmental constraints may also limit the amount of protection which technical mechanisms can provide. For example, there is little point in using an anonymous digital cash system to purchase goods over the Internet, if the purchaser’s identity is given away in the headers of his email message or if the goods are shipped to his home address.

E. Implementation considerations

The robustness of embedding and retrieving algorithms and their supporting protocols is not the only issue. Most real attacks on fielded cryptographic systems have come from the opportunistic exploitation of loopholes that were
We cannot expect copyright marking systems to be any different and the pattern was followed in the first attack to be made available on the Internet against one of the most widely used picture marking schemes. This attack exploited weaknesses in the implementation rather than in the underlying marking algorithms, even although these are weak (the marks can be removed with StirMark).

Each user has an ID and a two-digit password, which are issued when he registers with the marking service and pays a subscription. The correspondence between IDs and passwords is checked using obsolete software and, although the passwords are short enough to be found by trial and error, the published attack first uses a debugger to break into the software and disable the password checking mechanism. As IDs are public, either password search or disassembly enables any user to be impersonated.

A deeper examination of the program allows a villain to change the ID, and thus the copyright mark, of an already marked image as well as the type of use (such as adult versus general public content). Before embedding a mark, the program checks whether there is already a mark in the picture, but this check can be bypassed fairly easily using the debugger with the result that it is possible to overwrite any existing mark and replace it with another one.

Exhaustive search for the personal code can be prevented without difficulty, but there is no obvious solution to the disassembly attack. If tamper resistant software cannot give enough protection, then one can always have an online system in which each user shares a secret stege-key with a trusted party and uses this key to embed some kind of digital signature. Observe that there are two separate keyed operations here; the authentication (such as a digital signature) and the embedding or hiding operation.

Although we can do public key steganography – hiding information using a public key so that only someone with the corresponding private key can detect its existence – we still do not know how to do the hiding equivalent of a digital signature; that is, to enable someone with a private key to embed marks in such a way that anyone with the corresponding public key can read them but not remove them. Some attempts to create such watermarks can be found in [120]. But unless we have some new ideas, we appear compelled to use either a central ‘mark reading’ service or a tamper-resistant implementation, just as cryptography required either central notarisation or tamper-evident devices to provide a non-repudiation service in the days before the invention of digital signatures.

However, there is one general attack on tamper-resistant mark readers due to Cox et al. [122]. The idea is to explore, pixel by pixel, an image at the boundary where the detector changes from ‘mark absent’ to ‘mark present’ and iteratively construct an acceptable image in which the mark is not detected. Of course, with a programmable tamper-proof processor, one can limit the number of variants of a given picture for which an answer will be given, and the same holds for a central mark reading service. But in the absence of physically protected state, it is unclear how this attack can be blocked.

V. A basic theory of steganography

This leads naturally to the question of whether we can develop a comprehensive theory of information hiding, in the sense that Shannon provided us with a theory of secrecy systems [122] and Simmons of authentication systems [123]. Quite apart from intellectual curiosity, there is a strong practical reason to seek constructions whose security is mathematically provable. This is because copyright protection mechanisms may be subjected to attack over an extraordinarily long period of time. Copyright subsists for typically 50–70 years after the death of the artist, depending on the country and the medium; this means that mechanisms fielded today might be attacked using the resources available in a hundred years’ time. Where cryptographic systems need to provide such guarantees, as in espionage, it is common to use a one-time pad because we can prove that the secrecy of this system is independent of the computational power available to the attacker. Is it possible to get such a guarantee for an information hiding system?

A. Early results

An important step in developing a theory of a subject is to clarify the definitions. Intuitively, the purpose of steganography is to set up a secret communication path between two parties such that any person in the middle cannot detect its existence; the attacker should not gain any information about the embedded data by simply looking at cover-text or stego-text. This was first formalised by Simmons in 1983 as the ‘prisoners’ problem’ [124]. Alice and Bob are in jail and wish to prepare an escape plan. The problem is that all their communications are arbitrated by the warden Willie. If Willie sees any ciphertext in their messages, he will frustrate them by putting them into solitary confinement. So Alice and Bob must find a way to exchange hidden messages.

Simmons showed that such a channel exists in certain digital signature schemes: the random message key used in these schemes can be manipulated to contain short messages. This exploitation of existing randomness means that the message cannot even in principle be detected and so Simmons called the technique the ‘subliminal channel’. The history of the subliminal channel is described in [125], while further results may be found in [123], [126], [127].

In the general case of steganography, where Willie is allowed to modify the information flow between Alice and Bob, he is called an active warden; but if he can only observe it he is called a passive warden. Further studies showed that public key steganography is possible (in this model, Alice and Bob did not exchange secrets before going to jail, but have public keys known to each other) – although the presence of an active warden makes public key steganography more difficult [129].

This difficulty led to the introduction, in [130], of the supraliminal channel, which is a very low bandwidth chan-
nel that Willie cannot afford to modify as it uses the most perceptually significant components of the cover object as a means of transmission. For example, a prisoner might write a short story in which the message is encoded in the succession of towns or other locations at which the action takes place. Details of these locations can be very thoroughly woven into the plot, so it becomes in practice impossible for Willie to alter the message – he must either allow the message through or censor it. The effect of this technique is to turn an active warden into a passive one. The same effect may be obtained if the communicating parties are allowed to use a digital signature scheme.

B. The general role of randomness

Raw media data rates do not necessarily represent information rates. Analog values are quantised to \( n \) bits giving, for instance, a data rate of 16 bit/sample for audio or 8 bit/pixel for monochrome images. The average information rate is given by their entropy; indeed, the entropy of monochrome images is generally around 4–6 bits per pixel. This immediately suggests the use of this difference to hide information. So if \( C \) is the cover-text and \( E \) the embedded text, transmitted on a perfect \( n \) bit channel, one would have: \( H(E) \leq n - H(C) \) bit/pixel, so all the gain provided by compression is used for hiding. One could also take into account the the stego-text \( S \) and impose the constraint that no information is given about \( E \), even knowing \( S \) and \( C_k \) (a part of \( C \) typically the natural noise of the cover-text): the transinformation should be zero \( T(E;C_k,S) = 0 \). In this case, it can be shown that \( H(E) \leq H(C_k|S) \) \( [131] \). So the rate at which one can embed ciphertext in a cover-object is bounded by the opponent’s uncertainty about the cover-text given knowledge of stego-text. But this gives an upper bound on the stego-capacity of a channel when for a provably secure system we need a lower bound. In fact all the theoretical bounds known to us are of this kind. In addition, the opponent’s uncertainty and thus the capacity might asymptotically be zero, as was noted in the context of covert channels \( [132] \).

This also highlights the fact that steganography is much more dependent on our understanding of the information sources involved than cryptography is, which helps explain why we do not have any lower bounds on capacity for embedding data in general sources. It is also worth noting that if we had a source which we understood completely and so could compress perfectly, then we could simply subject the embedded data to our decompression algorithm and send it as the stego-text directly. Thus steganography would either be trivial or impossible depending on the system \( [119] \).

Another way of getting round this problem is to take advantage of the natural noise of the cover-text. Where this can be identified, it can be replaced by the embedded data (which we can assume has been encrypted and is thus indistinguishable from random noise). This is the philosophy behind some steganographic systems \( [133], [134], [60] \) and early image marking systems \( [22] \) (it may not work if the image is computer generated and thus has very smooth colour gradations). It can also be applied to audio \( [51] \). Randomisation is very important because simple replacement of the least significant bit causes an audible modification of the signal \( [51] \). So a subset of modifiable bits is chosen and the embedding density depends on the observed statistics of the cover-signal \( [?] \) or on its psychoacoustic properties \( [51] \).

It is also possible to exploit noise elsewhere in the system. For example, one might add small errors by tweaking some bits at the physical or data link layer and hope that error correction mechanisms would prevent anyone reading the message from noticing anything. This approach would usually fall foul of Kerckhoff’s principle that the mechanism is known to the opponent, but in some applications it can be effective \( [130] \).

A more interesting way of embedding information is to change the parameters of the source encoding. An example is given by a marking technique proposed for DVD. The encoder of the MPEG stream has many choices of how the image can be encoded, based on the trade-off between good compression and good quality – each choice conveys one or more bits. Such schemes trade expensive marking techniques for inexpensive mark detection; they may be an alternative to signature marks in digital TV where the cost of the consumer equipment is all-important \( [137] \).

Finally, in case the reader should think that there is anything new under the sun, consider two interpretations of a Beethoven symphony, one by Karajan the other one by Bernstein. These are very similar, but also dramatically different. They might even be considered to be different encodings, and musicologists hope to eventually discriminate between them automatically.

C. Robust marking systems

In the absence of a useful theory of information hiding, we can ask the practical question of what makes a marking scheme robust. This is in some ways a simpler problem (everyone might know that a video is watermarked, but so long as the mark is unobtrusive this may not matter) and in other ways a harder one (the warden is guaranteed to be active, as the pirate will try to erase marks).

As a working definition, we mean by a robust marking system one with the following properties:

- Marks should not degrade the perceived quality of the work. This immediately implies the need for a good quality metric. In the context of images, pixel based metrics are not satisfactory, and better measures based on perceptual models can be used \( [108], [138] \).
- Detecting the presence and/or value of a mark should require knowledge of a secret.
- If multiple marks are inserted in a single object, then they should not interfere with each other; moreover if different copies of an object are distributed with different marks, then different users should not be able to process their copies in order to generate a new copy that identifies none of them.
- The mark should survive all attacks that do not degrade the work’s perceived quality, including resampling.
re-quantisation, dithering, compression and especially combinations of these.

Requirements similar to these are found, for example, in a recent call for proposals from the music industry [139]. However, as we have shown with our attacks, there are at present few marking schemes, whether in the research literature or on commercial sale, that are robust against attacks involving carefully chosen distortions. Vendors when pressed claim that their systems will withstand most attacks but cannot reasonably be engineered to survive sophisticated ones. However, in the experience of a number of industries, it is ‘a wrong idea that high technology serves as a barrier to piracy or copyright theft: one should never underestimate the technical capability of copyright thieves’ [140].

Our current opinion is that most applications have a fairly sharp trade-off between robustness and data rate which may prevent any single marking scheme meeting the needs of all applications. However we do not see this as a counsel of despair. The marking problem has so far been over-abstracted; there is not one ‘marking problem’ but a whole constellation of them. Most real applications do not require all of the properties in the above list. For example, when monitoring radio transmissions to ensure that advert have been played as contracted, we only require enough resistance to distortion to deal with naturally occurring effects and prevent transfer of marks from one advert to another [141]; and where our concern is to make proprietary images available to scholars, as in the Vatican Library Accessible Worldwide project, IBM came up with a simple solution using visible watermarks – which leave the the documents still perfectly suitable for research purposes but discourage illegal publication for profit [11].

VI. Conclusion

In this paper we gave an overview of information hiding in general and steganography in particular. We looked at a range of applications and tried to place the various techniques in historical context in order to elucidate the relationships between them, as many recently proposed systems have failed to learn from historical experience.

We then described a number of attacks on information hiding systems, which between them demolish most of the current contenders in the copyright marking business. We have described a tool, StirMark, which breaks many of them by adding sub-perceptual distortion; and we have described a custom attack on echo hiding.

This led us to a discussion of marking in general. We described some of the problems in constructing a general theory and the practical requirements that marking schemes and steganographic systems may have to meet. We advanced the suggestion that it is impractical to demand that any one marking scheme satisfy all of these requirements simultaneously; that is, that ‘the marking problem,’ as sometimes described in the literature, is over-specified.

That does not of course mean that particular marking problems are insoluble. Both historical precedent and recent innovation provide us with a wide range of tools, which if applied intelligently should be sufficient to solve most of the problems that we meet in practice.

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